| **No.** | **Suggested Solutions** |  |
| --- | --- | --- |
| **1**            *O* |  |  |
| **2** | Let *A*, *C* and *S* be the number of adults, children and senior citizens in the local family.  ------------- (1)  ----------- (2)  ------------------------------- (3)  By GC,    ,  The family consists of 8 adults, 3 children and 2 senior citizens. |  |
| **3(i)** | Differentiating with respect to , |  |
| **3**  **(ii)** | For tangent to the curve to be parallel to the *x*-axis,      For all  Hence,    for all  Therefore, there is no point where the tangent is parallel to the *x*-axis. |  |
| **4(i)** | Common point *P*,  are collinear  **Alternative method**    is a point on line passing through  and |  |
| **4(ii)** | Since    is the length of projection of  onto |  |
| **4(iii)** |  |  |
| **5(i)** | *R*2  *R*1  *x*  *y*  *y* = e  *y* = – e  1  – 1  *P*  *y* =  Since  is an odd function, i.e. symmetrical about the origin,  Area of *R* |  |
| **5**  **(ii)** | *R*2  *R*1  *x*  *y*  *y* = e  *y* = – e  1  – 1  *P*  *y* =  Volume generated by *R*    **Method 2**  Volume generated by *R*    **Method 3**  Volume generated by *R* |  |
| **6(i)** |  |  |
| **6(ii)** | Since  is a constant  This is a Geometric Series with common ratio . |  |
| **6(iii)** | Since  Sum to infinity exits. |  |
| **7(i)** | When *C* crosses the *x*-axis ,    since |  |

|  |  |  |
| --- | --- | --- |
| **7**  **(ii)** | *y*  *x*    (7.85, 0)      (1.27, 0) |  |
| **7**  **(iii)** | ,      Equation of tangent at *P* : |  |
| **7**  **(iv)** | *y*  *x*  *O*  1  *Q*(1,1)  Tangent parallel to the *y*-axis:  is undefined  at  , *Q* |  |
|  | **Method 1**: Using parametric equations  Required Area  =  =  = 0.087955042  = 0.0880 units2 (3 s.f.) |  |
|  | **Method 2**: Using Cartesian equation  ,    Required Area  =  =  = 0.087954559  = 0.0880 units2 (3 s.f.) |  |
| **8(i)** | **Method 1**      Since g is self-inverse,    Using GC, |  |
|  | **Method 2**  Let      Since g is self-inverse, |  |
|  | **Method 3**    Since  is the horizontal asymptote, by symmetry  is the vertical asymptote. |  |
| **8**  **(ii)** | Since |  |
| **8**  **(iii)** | (2,7)  *y* = h(*x*)  *x*  *y*  C:\Users\S77313~1\AppData\Local\Temp\Texas Instruments\TI-SmartView CE for the TI-84 Plus Family\Capture1-1631189894265.png  Since  ,        Since  , |  |

|  |  |  |
| --- | --- | --- |
| **8**  **(iv)** | 0        and  Since  exits |  |
| **9(i)** | Method 1: (solve algebraically)  When ,    Method 2: (solve graphically using GC)  Using GC, |  |

|  |  |  |
| --- | --- | --- |
| **9(ii)** | *y*                *x*    *C*    *OO* |  |
| **9**  **(iii)** | Method 1: [using graph in part (ii)]    *y*        *x*  *C*    *OO*  from graph in (ii),  or  Method 2: (solve inequality directly)        Use simplified form after long division from part (ii)  or |  |
| **9**  **(iv)** | Replace *x* with  in (iii):  10  4  –2  +  +  –  –      Hence using result in (iii):  or  Method 1: (using graph of )  *y*  *x*  *O*    10  4  –2        From graph,  or  Note that  is also a solution to  Hence the final solution is  or  or  Method 2: (solving algebraically)  For :  and      or  …(1) and  or  …(2)  Combining (1) and (2), ie. taking intersection:  0    +  –  –  0    +  –  +  Hence  or  (\*)  0      (1)  (2)  Intersection of (1) and (2)  For :      (\*\*)  0    +  –  –  Hence combining (\*) or (\*\*), ie. taking union:  0        or  or  Note that  is also a solution to  Hence the final solution is  or  or |  |
| **9**  **(v)** | From graph in (iii):  *y*        *x*  *C*    *OO*      Required area  Method 1: (using partial fractions)  Area    Let  Use simplified form after long division from part (ii)    When ,  When ,  Area          where , ,  Method 2: (using  and MF26)  Area  Use simplified form after long division from part (ii)      Can also use partial fractions:      From MF26  where , , |  |
| **10**  **(i)** |  |  |
| **10**  **(ii)** | Let  be the total cost of manufacturing the prototype. |  |
|  | For stationary value, |  |
|  | **Using 2nd Derivative Test**:    Thus, *C* is a minimum when . |  |
|  |  |  |
|  | **Using 1st Derivative Test**:   |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  | – | 0 | + | | Shape | \ | – | / | |  |
| **10**  **(iii)** | After 5 seconds, amount of perfume leaked is    Total volume of hemisphere is  < 7.5  After 5 seconds, perfume is in the bottom segment, i.e. cylinder.        Height of perfume decreases at . |  |
| **11(i)** | *C*(6,1,4)  *F*      (1,2,4)  Let the foot of perpendicular be *F*.  Since *F* lies on ,  for some    Since        **Alternative Solution** (Projection Vector Method)  *C*(6,1,4)  *F*      *P*(1,2,4) |  |
| **11**  **(ii)** | (1 d.p) |  |
| **11**  **(iii)** | 2  *A*  *B*  *D*  1  *A*  *D*  1  *B*  1    or |  |
|  | Since *D* lies on  and ,  for some  and    or |  |
|  | Since *D* lies on  and ,  for some      From GC,  or  or |  |
| **11**  **(iv)** | Direction vector of  = |  |
| **12**  **(i)** | Interest rate for 12 months = *p*%  Interest rate per month = |  |
| **12**  **(ii)** | |  |  | | --- | --- | | **Month** | **Outstanding Loan** | | 0 | *L* | | 1 |  | | 2 |  | | 3 |  | | *n* |  |   -------------------------- (\*)      (Shown) |  |
| **12**  **(iii)** | In 30 years, *n* = 360,        OR  Outstanding loan,  By GC, when ,  Check: When ,  Their monthly instalment is $2017.72. (nearest cent)  Or  Their monthly instalment is $2020. (3 s.f.) |  |
| **12**  **(iv)** | When ,  by GC table,  C:\Users\S84256~1\AppData\Local\Temp\Texas Instruments\TI-SmartView CE for the TI-84 Plus Family\Capture1-1628612929749.png  12 years 4 months  The couple will fully repay the loan on 1 December 2033.  Final repayment = $1245.38 |  |
| **12**  **(v)** | Hence,  , |  |
|  | **Alternative solution**      Hence,  , |  |
| **12**  **(vi)** | Let  Use GC directly:  C:\Users\S77313~1\AppData\Local\Temp\Texas Instruments\TI-SmartView CE for the TI-84 Plus Family\Capture2-1631238600330.png  C:\Users\S77313~1\AppData\Local\Temp\Texas Instruments\TI-SmartView CE for the TI-84 Plus Family\Capture3-1631238735467.png  C:\Users\S77313~1\AppData\Local\Temp\Texas Instruments\TI-SmartView CE for the TI-84 Plus Family\Capture4-1631238794385.png  Least number of months = 87  Or use    To be keyed into GC to solve |  |